ERROR–AWARE GIS AT WORK: REAL–WORLD APPLICATIONS OF THE DATA UNCERTAINTY ENGINE

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ABSTRACT:

The call for the development of geographical information systems that can handle uncertain data dates back at least twenty years. Many research groups have since worked on the design of an 'error-aware GIS', but very few – perhaps none – have reached the operational stage. Recently, the Data Uncertainty Engine (DUE) was developed to fill this gap. DUE assists users in the assessment of uncertainties in environmental data and stores these in a spatio-temporal database. Uncertain environmental data are represented in DUE as objects whose positions and attribute values may be uncertain. Uncertainty is quantified with probability distribution functions, both for numerical and categorical data types. Spatial correlation between the vertices that define an object's position or between the point values of a spatially distributed attribute at multiple locations is also accommodated for. Both expert judgement and sample data may be used to help estimate the parameters of the probability distributions. DUE must be used alongside a GIS because it cannot do standard GIS operations, such as spatial queries, overlay, or visualisation. It is an add-on that can read and write common formats and can generate multiple realisations (i.e., random draws from the probability distribution) of an uncertain object. The realisations may be used in a Monte Carlo uncertainty propagation analysis or for communicating uncertainty to a target audience with limited background in statistics. The use of DUE has now surpassed the development stage that illustrated its use with synthetic examples. It is now being used in large projects to solve real-world problems. In this paper, three examples are given. These are: (1) the modelling, storage and simulation of uncertain soil profile and pesticide properties in a project that analyses how uncertainty in these properties affects the accuracy of the estimated nationwide leaching of pesticides in the Netherlands; (2) the stochastic simulation of spatially correlated soil aluminium and iron content in a project that assesses the water quality of the river Regge catchment; and (3) the handling of positional uncertainty in the delineation of breeding bird areas in a spatial planning project in the South of the Netherlands.

1. INTRODUCTION

When data that are stored in spatial databases are inaccurate then the results of spatial analyses that use these data as input will be inaccurate too. The awareness that uncertainty propagates through spatial analyses and may produce poor results that lead to wrong decisions has triggered a lot of research on spatial accuracy assessment and data quality management in GIS (e.g., Heuvelink 1998, Shi et al. 2002). The ideal system that can cope with spatial data quality is that of an 'error-aware' GIS, which can assess and store the uncertainties associated with its data and can analyse and present the propagation of uncertainty in GIS analyses. Burrough (1992) sketched an 'intelligent' GIS that would have all of these capabilities and would in addition be able to advise users on how best to improve the quality of their results, either by using better models, collecting more or better data, or improving resolution. Duckham (2002) and Duckham and McCreadie (2002) laid down a design of how an 'error-aware' or 'error-sensitive' GIS should look like, but the concepts and ideas presented seem not to have made it to the operational stage. Karssenberg and De Jong (2005) extended the PCRaster software with uncertainty propagation functionality, but the focus of their work was on the implementation of Monte Carlo uncertainty propagation routines, not on the assessment and storage of spatial data accuracy.

Recently, the Data Uncertainty Engine (DUE) was developed to provide a tool that comes close to an operational 'error-aware' GIS (Brown and Heuvelink 2007, Heuvelink et al. 2007). DUE helps users to assess and store uncertainties in environmental data and provides functions to generate realisations (random draws) of uncertain data for visualisation of uncertainty and use in uncertainty propagation analyses.

DUE takes a probabilistic approach to represent uncertainty in the position and attribute values of spatial objects. The rationale behind this approach is that because of uncertainty, the true state of the environment is not precisely known, but only to some degree. In other words, it is based on the premise that the person or organisation is not completely ignorant but is able to characterise the state with a probability distribution function. For instance, the annual greenhouse gas emission of a country cannot be measured or calculated exactly and is therefore uncertain. However, expert judgement, measurements and models do provide the means to provide an estimate of the annual emission and quantify the associated estimation error. Thus, the annual emission may be represented by a probability distribution function (pdf), which essentially lists all possible outcomes and their associated probabilities.

In this paper we summarise the theory and concepts behind DUE, its functionality and user-interface, and illustrate its use in practice with three real-world examples. For a detailed account of DUE, see Brown and Heuvelink (2007).

2. PROBABILITY MODELS FOR POSITIONAL AND ATTRIBUTE UNCERTAINTY

2.1 Positional uncertainty

In order to describe the positional uncertainty of an environmental object, it is useful to classify objects by their primitive parts and by the types of movement they support under uncertainty. A first-order classification would include:

1. Objects that are single points (point objects);
2. Objects that comprise multiple points whose relative positions cannot change under uncertainty (rigid objects);
3. Objects that comprise multiple points whose relative position can vary under uncertainty (deformable objects).

The positional uncertainty of a point object always leads to a unitary shift in the object's position in the x- and y-direction (assuming two-dimensional space). The positional uncertainty of a rigid object comprises a uniform translation of its internal points and rotation of the object about an origin. By implication, positional uncertainty cannot alter the topology of a rigid object. In contrast, the topology of a deformable object may be altered and corrupted by positional uncertainty because the uncertainties in its primitive points are partially or completely independent of each other. Topological corruption of deformable objects can be prevented in practice by discarding random samples from the pdf whose topological relations are deemed invalid.

We develop a general probability model for each class of object distinguished above. Because of uncertainty, the 'true' value of the coordinates of a point object (e.g. x) is unknown and hence it is represented as a random variable X with a marginal probability distribution function F_X:

$$F_X(x) = P(X \le x) \tag{1}$$

where x is a real number and P is probability. The pdf F_X must be a continuous, non-decreasing, function, whose limit values are $F_X(-\infty)=0$ and $F_X(+\infty)=1$. Marginal distributions may be defined for both coordinates of an uncertain point object. When the uncertainties in the coordinates are statistically dependent, a multivariate or joint pdf is required:

$$F_{XY}(x, y) = P(X \le x, Y \le y) \tag{2}$$

When the uncertain coordinates are independent, the joint pdf is simply the product of the two marginal pdfs.

Rigid objects comprise multiple points whose internal angles and distances cannot change under uncertainty. However, the object may rotate, as well as shift, under uncertainty. In practice, the movement of a rigid object, and hence its positional uncertainty, is completely characterised by the translation and rotation of a single point associated with that object (e.g. its centroid). Thus, a joint pdf is required for the coordinates of the reference point together with the rotation angle.

Deformable objects comprise multiple points whose internal distances and angles can vary under uncertainty. As such, the positional uncertainty of a deformable object cannot be described with a simple translation and rotation of the object, but requires a separate pdf for each primitive point, together with the internal relations (correlations) between these points. Thus, for a two-dimensional object containing n primitive points, a 2n-dimensional pdf is required:

$$\begin{aligned} &F_{X_1Y_1 \ldots X_nY_n}(x_1, y_1, \ldots, x_n, y_n) = \\ &= P(X_1 \le x_1, Y_1 \le y_1, \ldots, X_n \le x_n, Y_n \le y_n) \end{aligned} \tag{3}$$

In practice, it is rarely realistic to derive Eqn. (3) as the product of n pdfs specified in Eqn. (2) because data collection and pre-processing will introduce statistical dependencies between points. For example, GPS surveys, georeferencing of remote sensing data, and manual digitising of paper maps will all introduce positive correlations between positional uncertainties (see also De Bruin et al. 2007).

2.2 Attribute uncertainty

In order to develop probability models for attribute uncertainty, it is useful to distinguish between the measurement scales of an attribute. Important classes of measurement scale are the continuous numerical scale (e.g. air humidity), the discrete numerical scale (e.g. bird counts) and the categorical scale (e.g. soil type). In addition, attributes can be constant in space or vary in space.

An uncertain continuous numerical constant C is characterised by its pdf:

$$F_C(c) = P(C \le c) \tag{4}$$

The pdf for a discrete numerical or categorical constant is:

$$F_C(c_i) = P(C = c_i) \tag{5}$$

where the c_i (i=1,..,m) are numbers or categories, respectively. Each of the $F_C(c_i)$ should be non-negative and the sum of all $F_C(c_i)$ should equal 1.

An uncertain continuous numerical variable V that varies in space is characterised by:

$$F_V(v_1, s_1 \ldots, v_n, s_n) = P(V(s_1) \le v_1, \ldots, V(s_n) \le v_n) \tag{6}$$

where the s_i are coordinates (i.e., s_i comprises x_i and y_i) and n may assume any integer value. F_V must be known for each and every combination of the v_i and s_i. The corresponding pdf for a discrete numerical or categorical variable that is spatially variable is:

$$F_V(v_1, s_1, \ldots, v_n, s_n) = P(V(s_1) = v_1, \ldots, V(s_n) = v_n) \tag{7}$$

where the v_i are integers or categories, respectively. The pdfs Simplifying assumptions are typically needed to estimate the pdfs (6) and (7) in practice. This is discussed in the next section.

2.3 Estimation of pdfs

The probability models defined in the previous section must be specified for each particular case. This typically involves a trade-off between the complexity of the pdf and the amount of information available to estimate it. A common assumption is that the pdf follows a simple shape, such as the 'Normal' or 'Exponential' distribution for continuous variables and the 'Poisson' or 'Binomial' for discrete and categorical variables. The parameters of these distributions (e.g. the mean and variance for the normal distribution) may be estimated from expert judgement or sample data (Brown and Heuvelink 2007). For some numerical variables, and for most categorical variables, an appropriate parametric shape may not be available. In that case, each possible outcome and its associated probability must simply be listed in a 'non-parametric' pdf.

For cases in which a parametric pdf is applied to an attribute that varies in space, some or all of the model parameters may vary in space. Furthermore, the uncertain attributes and the positional uncertainties of objects may be statistically dependent in space. For continuous numerical variables, quantification of the spatial dependence is not difficult when the normal distribution and second-order stationarity are assumed. The latter means that the associated pdf has a constant variance and a spatial correlation that only depends on the distance between locations (Heuvelink 1998, Section 5.2). In that case, the spatial dependence structure may be estimated with a simple function (i.e., a semivariogram or correlogram), for which common geostatistical procedures can be used (Goovaerts 1997). This approach also applies to the positional uncertainty of objects. For example, it may be sensible to assume that the correlation between the primitive points of a deformable object only depend on the Euclidean distance between them.

3. THE DATA UNCERTAINTY ENGINE

3.1 Overview and current functionality of DUE

The Data Uncertainty Engine (DUE) is a prototype software tool for assessing uncertainties in environmental data, storing them within a database, and for generating realisations of data to include in Monte Carlo uncertainty propagation studies (e.g. Heuvelink 1998, Van Niel et al. 2004, Karssenberg and De Jong 2005, Miehle et al. 2006). The software is intended for researchers and practitioners who understand the problems of uncertainty in spatial data and models but do not have the time or background in uncertainty methods to design their own study with more generic tools, such as R or Matlab.

The functionality currently supported by DUE includes:

- A conceptual framework for guiding an uncertainty assessment, which is implemented through a graphical user interface;
- The specification of a probability model for continuous numerical attributes, discrete numerical attributes and categorical attributes. The attributes may be constant or may vary in space. In fact, DUE also supports variables that vary in time. Multivariate pdfs can be defined for groups of attributes and for the coordinates of objects;
- Parametric pdfs for continuous (e.g. normal, lognormal, Weibull) and discrete numerical data (e.g. Poisson, binomial, uniform), with the option to define a non-parametric pdf for discrete numerical and categorical data;
- The use of expert judgement and/or sample data to help define a pdf. Sample data can be used to improve the accuracy and reduce the uncertainty of attributes by ensuring that each realisation reproduces the sample values (i.e. conditional simulation);
- The specification of correlations within a single object or attribute and cross-correlations between objects or attributes (only if the pdfs follow a joint normal distribution);
- Aggregation of (uncertain) attribute values to larger spatial scales, including aggregation from points to blocks;
- Efficient stochastic simulation from pdfs. An exact, and fast, simulation routine is used for correlated normal variables if the correlation matrix is sufficiently small. Otherwise, simulation is conducted using the sequential simulation algorithm (Goovaerts 1997). Sequential simulation relies on the Gstat executable (Pebesma 2004);
- Import from and export to file (with a limited range of formats), as well as a 'DUE-enabled' database;
- Use of the Java programming language, which is platform independent and may be executed on all operating systems that support a Java Virtual Machine;
- A resource for developers, which is extensively documented in HTML format;
- Release of the software under the General Public Licence; it is, therefore, free to use, modify and distribute.

3.2 Performing an uncertainty assessment with DUE

An uncertainty assessment with DUE is separated into five stages, namely:

1. Loading (and saving) data ("Input");
2. Identifying the causes or 'sources' of uncertainty ("Sources");
3. Defining an uncertainty model for the combined sources of uncertainty ("Model");
4. Reflecting on the quality or 'goodness' of the model ("Goodness");
5. Simulating from an uncertainty model for visualisation and Monte Carlo uncertainty propagation studies ("Output").

These stages are presented as 'tabbed windows' in DUE. An uncertainty assessment may involve sequentially navigating through these windows or entering at an arbitrary point, depending on the aims of a session, which may include assessing uncertainty, modifying or simulating from an existing uncertainty model. Stages 2 and 4 (describing the sources of uncertainty and assessing goodness) may be skipped, but are useful for structuring an uncertainty assessment and for quality control, respectively. A searchable library of uncertainty sources is provided for this purpose.

Figure 1 presents the "Input" window of DUE. As indicated above, data may be loaded into DUE from file or from a database, and are stored within DUE as objects, whose positions may be uncertain, and attributes, whose values may be uncertain. Once imported, an uncertainty model may be defined for the objects and attributes selected in the opening dialog. In the first window of the "Model" pane, an uncertainty model structure is chosen for the selected objects and attributes. If sample data are available, they are selected here. In the absence of sample data, an uncertainty model must be defined through expert judgement alone. If the uncertainties are assumed spatially correlated, then a correlation model must be defined in a subsequent window (Figure 2).

Once complete, an uncertainty model can be used to generate realisations of the uncertain objects and attributes in the "Output" window (Figure 3). In order to simulate from an uncertainty model, the output scale, the number of realisations and the location for writing data must be specified.

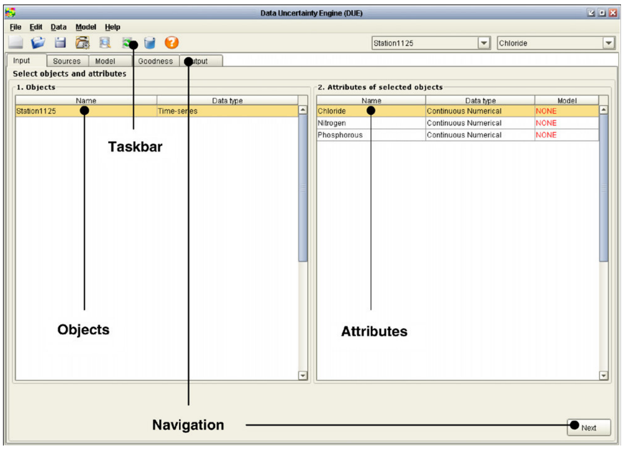

Figure 1. The "Input" window of DUE showing one imported object with three attributes.

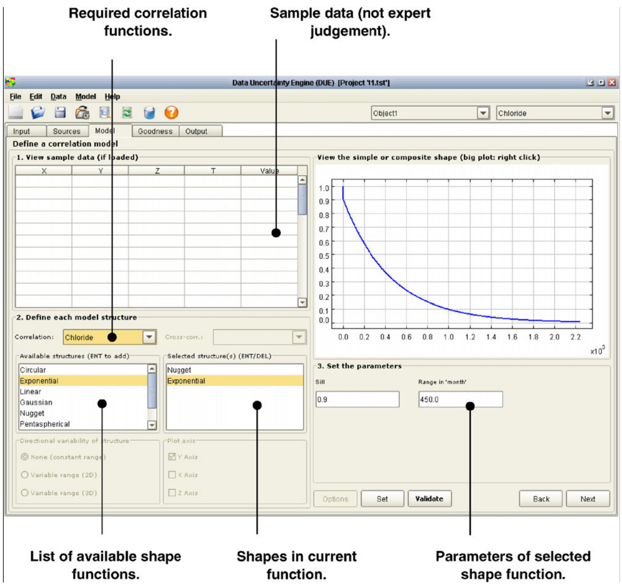

Figure 2. Defining a correlation function in DUE.

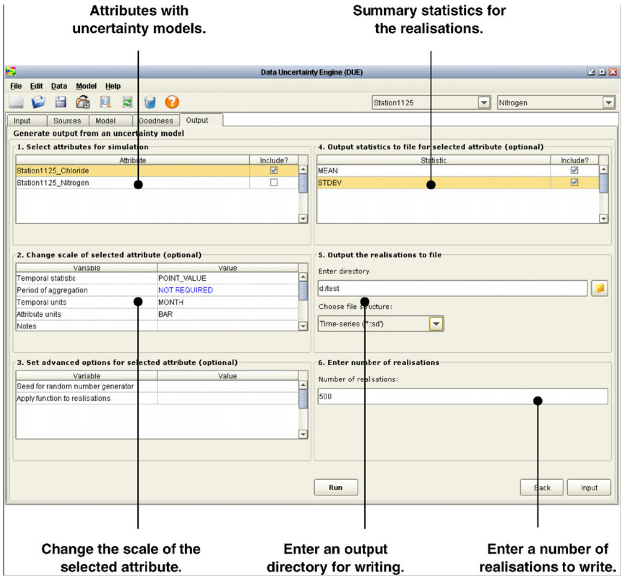

Figure 3. The "Output" window of DUE to generate realisations of uncertain objects.

4. THREE EXAMPLES OF REAL–WORLD APPLICATIONS OF DUE

4.1 Uncertainty analysis of the GeoPEARL pesticide leaching model

In the new Dutch decision tree for the evaluation of pesticide leaching to groundwater, data on the spatial distribution of soils are used by the GeoPEARL model to calculate the 90th percentile of the leaching concentration in the area of potential usage. Until recently it was not known to what extent the uncertainties in soil and pesticide properties propagate to the predicted leaching concentrations. Therefore, a study was set up to quantify the uncertainties in the soil and pesticide data and analyse their contribution to the uncertainty in the leaching concentrations (Van Den Berg et al. 2007). The contribution from uncertain soil properties (e.g. soil horizon thickness, texture, organic matter content hydraulic conductivity and water retention characteristics) was compared to that caused by uncertainties in the most important pesticide properties, i.e. the half-life of transformation in soil (DT50) and the coefficient of sorption on organic matter. Firstly, the uncertainties in the soil and pesticide properties were quantified. Next, a regular grid sample of points covering the whole of the agricultural area in the Netherlands was randomly selected. At the grid nodes, realisations from the probability distributions of uncertain inputs were generated and used as input to a Monte Carlo uncertainty propagation analysis. It turned out that uncertainties in DT50 and Kom contributed most to the uncertainty in the leaching concentrations.

In this study, DUE was used to generate 500 realisations of the uncertain pesticide properties for three representative substances. Extreme values in DT50 and Kom are common and both attributes were therefore represented by lognormal distributions. Literature studies on the variability in the rate of degradation and sorption coefficient of various pesticides in different soils revealed that the coefficient of variation was about 25 per cent for both attributes (Allen and Walker 1987). Mean values of the log-tranformed properties were also taken from the literature. Correlation between DT50 and Kom was ignored.

4.2 Uncertainty analysis of a regionalised water quality model

The influence of input data and parameter uncertainty of diffuse emission sources on the summer averaged phosphorus and nitrogen concentrations at the outlet of the Regge river catchment was analysed for 1999 (Bijlsma et al. 2006). The Regge is a small river running through the East of the Netherlands and a small part of Germany. The catchment area is about 1000 km^2. The soil type is mainly sandy and the main land use is a mixture of livestock and arable farming, with crops feeding the livestock and the livestock providing manure to the arable operations. The average phosphorus and nitrogen concentrations were computed with the NL-CAT model (Schoumans et al. 2005), which has a soil and groundwater flow module, a soil nutrient cycle and leaching module, a surface water flow module and a surface water quality module.

The uncertainty analysis focused on several important uncertainty sources. These were fertilizer application load, phosphorus background concentration of the groundwater, gas diffusion parameters in soil; and iron and aluminium content of the soil (0−120 cm). DUE was used to generate realisations of the iron and aluminium concentrations to be used in the Monte Carlo uncertainty propagation analysis. Using a dataset of 168 point observations, the mean, variance and correlogram of the iron and aluminium concentrations were determined and conditional sequential Gaussian simulation was used to generate 400 realisations of both attributes. Because iron and aluminium concentration are correlated with soil type, the statistical model that was designed allowed different means and standard deviations per soil type. By subtracting these means from the observed values and dividing by the standard deviation, residuals at the 168 points were obtained, for which the spatial correlation structure was assessed in DUE. Although in principle all steps could have been done with DUE, it turned out to be more efficient to run the standardisation and destandardisation outside DUE and use DUE only to characterise and simulate the spatially correlated residual.

4.3 Handling positional uncertainty in a spatial planning project

The Dutch Ministry of Housing, Spatial Planning and the Environment aims to have digital, exchangeable and comparable spatial plans. The new spatial planning law therefore compels the use and development of digital and exchangeable plans. However, first experiences show that uncertainty is an important factor that hinders the comparability of spatial plans. Therefore, a project was initiated that addresses uncertainty in spatial planning and aims to provide tools for handling uncertainty in spatial planning, such that the transparency of spatial planning processes is improved (Vullings et al. 2007). Based on a general taxonomy for uncertainty (Fisher et al. 2005), a taxonomy of uncertainty for spatial planning was developed. In this taxonomy, the sources of uncertainty in plans, processes and procedures and their possible solutions are visualised. One of the many aspects that were considered was the positional uncertainty in delineated breeding bird areas. In conventional analyses, these areas have exact spatial boundaries but in reality there is large uncertainty about where a breeding bird area begins and ends. Therefore, one of the goals of the project was to analyse how positional uncertainty about breeding bird areas would affect the outcome of a complex spatial planning process for the province of Noord-Brabant.

The shape files of the breeding bird areas were loaded into DUE and a statistical model (pdf) of the uncertain position of the polygons was built. Because no training data were available, the model was entirely based on expert judgement. Realistic values for the standard deviations in the x- and y-coordinates and the correlation between neighbouring vertices of each polygon were defined. It was assumed that the standard deviations in the x- and y-directions were equal. The correlation was assumed to depend only on the distance between points, whereby a Gaussian-shaped correlogram was used. DUE was then used to simulate 100 possible realities of the delineated breeding bird areas. Several of the simulated areas were topologically corrupt. These realisations were discarded and replaced by new simulations. After running the entire automated spatial planning process, it turned out that the positional uncertainty about the breeding bird areas marginally affected the final plan.

5. DISCUSSION AND CONCLUSIONS

DUE was developed to satisfy the need for an error-aware GIS. Even though the theoretical concepts behind the tool may be sound and represent the state-of-the-art, the usefulness of the tool in practice can only prove itself by application to real-world problems. In this paper, we have briefly reported on the use of DUE in three projects that applied uncertainty analysis in different settings. Although DUE was able to do the job it was asked to do in all three cases, the experiences were not entirely positive. It turned out that the flexibility of DUE was somewhat limited and that it could not store and simulate all uncertain objects considered in the analyses. Some of the uncertain inputs had to be simulated outside DUE in a more flexible programming environment (i.e., Matlab). Also, the user-interfacing of DUE may still be improved and the tool does not yet seem to be entirely free of bugs.

In spite of these shortcomings, there is no doubt that DUE has the potential to be the first error-aware GIS that is used by GIS practitioners at large. However, for this to happen the prototype DUE must be further elaborated and improved. Feedback from current users is important. In this respect, it is encouraging that DUE is used in other projects as well (e.g. De Bruin et al. 2007) and that it is part of the MSc GeoInformation programme of Wageningen University. Incorporating DUE and the underlying theories and methods in teaching is important also because it ensures that the next generation of GIS specialists is aware of spatial data quality issues and knows of the tools that can assist GIS users in managing spatial accuracy.

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